# Fuzzy Model–based H∞ Control for Nonlinear Cooperative Adaptive Cruise Control

Wei Yue<sup>1, a</sup>, Wang Liyuan<sup>2, b</sup>

<sup>1</sup>College of Marine Electrical Engineering, Dalian Maritime University, Dalian, China <sup>2</sup>College of Mechanical and Electronic Engineering, Dalian Minzu University <sup>a</sup>yuewei811010@163.com, <sup>b</sup>wangliyuandmu@163.com

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*Abstract:* This paper presents a fuzzy model-based control algorithm for nonlinear interconnected cooperative adaptive cruise control system (CACCs). Firstly, a nonlinear model of the CACCs' longitudinal movement is replaced by an equivalent Takagi-Sugeno type fuzzy model. Then, this paper designs a decentralized state feedback fuzzy controller to override the external disturbances such that the CACCs can achieve a good performance. Finally, the asymptotic stability and string stability of the nonlinear interconnected CACCs is guaranteed. The effectiveness of the proposed method is demonstrated by simulations.

## **1. Introduction**

In recent years, urban traffic have been rapidly developed which caused a huge burden on the existing transportation system. In this respect, many researchers have devoted to develop the intelligent traffic systems [1], the goal is to find an effective way to reduce the traffic congestion. In the recent study, cooperative adaptive cruise control (CACC) was regarded as the most effective technology in the field of intelligent transportation system [2]. Unfortunately, the CACCs inevitably suffer from communication constraints, complex dynamics, and unknown interference from the outside world such as winds and roads [3, 4], thus, leading to a difficult to design effective control strategy. But, quite apart from that, vehicles in a CACC are dynamically coupled by controller structure, position and velocity of one vehicle may affect the others or even amplify as they propagate upstream along the CACCs and destroy the entire performance.

Previous works pertaining to the CACC mainly about obtaining a controller and spacing scheme to adjust the speed of vehicles [5]. Generally speaking, two typical types of spacing schemes which are widely used in cooperative control of vehicles, such as, the constant-spacing scheme and the time headway spacing scheme. The constant time headway spacing scheme have successfully applied to adaptive cruise control (ACC) [6-7]. The constant-spacing scheme is widely used for autonomous platoon control, which we focus in this research.

However, the existing controllers' designs for CACC are at least a lack of two considerations. Firstly, a simple linear vehicle model was frequently used in [8], which is usually very difficult to apply to practice, especially when considered with the effect of interconnected term. The combined interconnected is another aspect that may it may increase the difficulty of the design of the controller since interconnected term is the major factor in string unstable. The problem of interconnected has been researched by many scholars under different frameworks, such as in [9] the interconnected from the preceding vehicle was treat as an disturbance under which the string unstable can be avoided, and in [10] by utilizing inclusion principle to decoupling the interconnected term. However, these researches are derived based on a linearized vehicle dynamics model, which are not sufficient for achieving more stringent performance requirement for nonlinear CACCs. To the best of the authors' knowledge, systematic analysis considerations the desired CACCs performance, interconnected dynamics are usually omitted.

The paper is organized as follows. In Section II, a nonlinear CACC model is built. In Section III, a fuzzy model based controller design procedure is investigated for dealing with the nonlinearity interconnected term. The issue of string stability is also analyzed in this section. In Section IV, simulation examples are provided to demonstrate the design procedures. The concluding is made in Section V.

### 2. Problem Formulation

Consider a CACC system consisting of *N* vehicles running in a horizontal environment. Each vehicle transmits its acceleration to its follower via a wireless communication channel. The distance and relative velocity between two adjacent vehicles are measured by on-board sensors.

## 2.1 CACCs modeling

The spacing errors of the two consecutive vehicles are defined as:

$$\delta_i = z_{i-1} - z_i - L_i - hv_i, \qquad (1)$$

where *h* is the time gap,  $z_{i-1}$  and  $z_i$  denote the position of two consecutive vehicles,  $v_i$  and  $L_i$  denotes the velocity and length of the vehicle.

The dynamics of the *i*th following vehicle formulated as follows:

$$a_{i}(t) = \frac{1}{m_{i}} (F_{i}^{e}(t) - F_{i}^{w}(t) - d_{i}^{m}(t) - m_{i}g\sin\theta_{i}), \qquad (2)$$

where  $d_i^m$  is the mechanical drag,  $F_i^e = m_i \varsigma_i$  is the force produced by the vehicle's engine,  $m_i g \sin \theta_i$  denotes the component of the vehicle weight, and  $m_i$  denotes the vehicle's mass, g denotes the acceleration of gravity, and  $\theta_i$  denotes the angle between the road surface and a horizontal plane;  $F_i^w$  specific the force due to the air resistance and defined as,

$$F_{i}^{w}(t) = \frac{\sigma A_{i} c_{di}}{2} (v_{i}(t) + v_{wind}(t))^{2} \operatorname{sgn}(v_{i}(t) + v_{wind}(t))$$
(3)

where  $\sigma$  denotes the specific mass of the air,  $A_i$  is the cross-sectional area,  $c_{di}$  is the drag coefficient,  $v_{wind}$  is the velocity of the wind gust.  $F_i^e = m_i \varsigma_i$  is the engine force of vehicle *i*, and the engine dynamics satisfies [11], which can be model as

$$\dot{\varsigma}_i(t) = -\frac{1}{\tau_i}\varsigma_i(t) + \frac{1}{m_i\tau_i}u_i(t)$$
(4)

where  $\tau_i$  denotes the vehicle's engine time-constant,  $u_i(t)$  denotes the throttle input to the

vehicle's engine.

According to [12], the angle  $\theta_i$  which is correlated in time and expressed as

$$\dot{\theta}_i(t) = -\alpha \theta_i(t) + w_i(t) \tag{5}$$

where  $\alpha$  is the reciprocal of the time constant and  $w_i(t)$  is random road induced vibrations for the *i*th vehicle.

From (2) and (3), if the wind gust  $v_{wind}(t) = 0$  and the vehicle travels in the same direction at all time  $sgn(v_i(t) + v_{wind}(t)) = 1$ , we can get,

$$\varsigma_i(t) = a_i - \frac{\sigma A_i c_{di}}{2m_i} v_i^2(t) - \frac{d_i^m(t)}{m_i} - g\sin\theta_i(t)$$
(6)

Substituting the expression for  $\varsigma_i(t)$  from (6) in the engine dynamics in (4), we obtain,

$$\dot{\varsigma}_i(t) = -\frac{1}{\tau_i} a_i - \frac{\sigma A_i c_{di}}{2\tau_i m_i} v_i^2(t) - \frac{d_i^m(t)}{\tau_i m_i}$$

$$-\frac{g}{\tau_i}\sin\theta_i + \frac{1}{m_i\tau_i}u_i(t) \tag{7}$$

Differentiating both sides of (6) and substituting the expression for  $\dot{\zeta}_i(t)$  from (7) we get,

$$\dot{a}_i(t) = -\frac{1}{\tau_i}a_i - \frac{K_i}{\tau_i m_i}v_i(t)a_i(t) - \frac{K_i}{2\tau_i m_i}v_i^2(t) - \frac{d_i^m(t)}{\tau_i m_i} + \alpha_g \theta_i(t)\cos\theta_i(t) - \frac{g}{\tau_i}\sin\theta_i(t)$$

$$-gw(t)\cos\theta_i(t) + \frac{1}{m_i\tau_i}u_i(t)$$
(8)

where  $K_i = \sigma A_i c_{di}$ . By combining the dynamics of the vehicle (8) and (1), we derived the following state space equation for the *i*th vehicle in CACCs,

$$\dot{x}_i(t) = f_i(x_i(t)) + g_i(x_i(t))u_i(t) + \Delta_i(t, x_i(t), x_{i-1}(t))$$

$$+d_i(t)$$
 (9)

where  $x_i(t) = [\delta_i \quad v_i \quad \theta_i \quad a_i]^T$  ( $z_0 = 0$  in  $\delta_1$ ) denotes the state of the system,  $d_i(t)$  denotes the disturbance.

$$f_i(x_i(t)) = \begin{bmatrix} -v_i - ha_i & a_i & -\alpha\theta_i(t) & -\frac{K_i}{\tau_i m_i}v_i(t)a_i(t) \\ -\frac{K_i}{2\tau_i m_i}v_i^2(t) + \alpha g\theta_i(t)\cos\theta_i(t) - \frac{g}{\tau_i}\sin\theta_i(t) - gw(t)\cos\theta_i(t) \end{bmatrix}^T,$$
  

$$g_i(x_i(t)) = \begin{bmatrix} 0 & 0 & 1/m_i\tau_i \end{bmatrix}^T,$$
  

$$\Delta_i(t, x_i(t), x_{i-1}(t)) = \begin{bmatrix} v_{i-1} & 0 & 0 & -\frac{d_i^m(t)}{\tau_i m_i} \end{bmatrix}^T \text{ denotes the nonlinear coupled term.}$$

A fuzzy CACCs dynamic model was proposed based on Takagi-Sugeno [13] to represent locally linear input/output relations for the nonlinear CACC system (2). We used fuzzy If-Then rules to

describe the fuzzy CACC model and applied it to deal with the problem of nonlinear interconnected CACCs. The *j*th rule of this fuzzy CACC model is proposed as,

Plant Rule *j*:

If  $\theta_{i1}(t)$  is  $W_{i1}^{j}(t)$ ,..., and  $\theta_{ip}(t)$  is  $W_{ip}^{j}(t)$ , Then

 $\dot{x}_i(t) = A_{ii}x_i(t) + B_{ii}u_i(t) + A_{i(i-1)i}x_{i-1}(t) + d_i(t)$ (10)

for j=1,2,...,L where  $W_{ip}^{j}(t)$  is the fuzzy sets, *L* is the number of If-Then rules, the matrices  $A_{ij}$ ,  $B_{ij}$ ,  $A_{i(i-1)j}$  and  $D_{ij}$  are of appropriate dimensions, and  $d_i(t)$  is the external disturbance for the *i*th vehicle.

Denote  $\theta_i(t) = [\theta_{i1}(t), \theta_{i2}(t), \dots, \theta_{ip}(t)]^T$  is the premise variable. By using the center-average defuzzifier and product inference, the overall fuzzy CACC system (9) can be rearranged as the following form,

$$\dot{x}_{i}(t) = \sum_{j=1}^{L} h_{ij}(\theta_{i}(t)) \Big[ A_{ij} x_{i}(t) + B_{ij} u_{i}(t) + A_{i(i-1)j} x_{i-1}(t) \Big] + d_{i}(t)$$
(11)

where  $h_{ij}(\theta_i(t)) = \mu_{ij}(\theta_i(t)) / \sum_{j=1}^{L} \mu_{ij}(\theta_i(t))$  with  $\mu_{ij}(\theta_i(t)) = \prod_{q=1}^{p} W_{iq}^j(\theta_{iq}(t))$ .

where  $W_{iq}^{j}(\theta_{iq}(t))$  is the grade of membership of  $\theta_{iq}(t)$  in  $W_{iq}^{j}$  [14].

We assume  $\mu_{ij}(\theta_i(t)) \ge 0$ , and  $\sum_{j=1}^{L} \mu_{ij}(\theta_i(t)) > 0$  for all t > 0.

Therefore, we get

$$h_{ij}(\theta_i(t)) \ge 0, \ \sum_{j=1}^L h_{ij}(\theta_i(t)) = 1.$$
 (12)

Suppose the following fuzzy controller is employed to deal with the nonlinear CACC system (11),

Control Rule s: If  $\theta_{i1}(t)$  is  $W_{i1}^{j}(t), \dots$ , and  $\theta_{ip}(t)$  is  $W_{ip}^{j}(t)$ ,

Then  $u_i(t) = K_{is}x_i(t)$ 

Hence, the fuzzy decentralized controller can be designed as

$$u_{i}(t) = \sum_{s=1}^{L} h_{is}(\theta_{i}(t)) K_{is} x_{i}(t)$$
(13)

Substituting (13) into (11) yields the closed-loop CACC system as following

$$\dot{x}_{i}(t) = \sum_{j=1}^{L} \sum_{s=1}^{L} h_{ij}(\theta_{i}(t)) h_{is}(\theta_{i}(t)) \Big[ (A_{ij} + B_{ij}K_{is}) x_{i}(t) \Big]$$

$$+A_{i(i-1)j}x_{i-1}(t)]+d_{i}(t)$$
(14)

## 2.2 The objective

The objective of this research is to design a  $H_{\infty}$  decentralized fuzzy model based method for the CACCs to meet the criterions as:

1) Asymptotic stability: The spacing error and velocity error of each vehicle approach to zero when the CACCs moving with a constant velocity.

2) String stability: The oscillations are not amplifying with vehicle index caused by any maneuver of the lead vehicle.

Besides the above stability criterions, the  $H_{\infty}$  performance related to  $x_i(t)$  requirement to be guaranteed as following,

$$\int_{0}^{t_{f}} x_{i}^{T}(t) Q_{i} x_{i}(t) dt \leq \rho^{2} \int_{0}^{t_{f}} d_{i}^{T}(t) d_{i}(t) dt , \qquad (15)$$

where  $Q_i$  is given positive definite symmetric matrices,  $t_f$  is the terminal time of control,  $\rho$  is a prescribed attenuation level.

**Remark 1.** Note that the meaning of (14) is the effect of any  $d_i(t)$  on  $x_i(t)$  must be attenuated below a desired level  $\rho$ , i.e., the  $L_2$  gain from  $d_i(t)$  to  $x_i(t)$  must be equal to or less than a prescribed value  $\rho^2$ . The  $H_{\infty}$  performance with a prescribed attenuation level is useful for a robust controller design without knowledge of  $d_i(t)$ ..

### **3. Fuzzy Model Based H** $\infty$ Controller Design

Based on the model and preliminaries given in the above section, a sufficient condition is given for the CACCs to ensure that all the vehicles in the string are asymptotically stable with the effect of external disturbance.

Our first result is given in the following theorem.

**Theorem 1.** For the closed loop CACC system in (14), if  $P_i = P_i^T > 0$  is a common solution of the following linear matrix inequality

$$\begin{bmatrix} (A_{ij} + B_{ij}K_{is})^T P_i + P_i(A_{ij} + B_{ij}K_{is}) & P_iA_{i(i-1)j} \\ A_{i(i-1)j}^T P_i & 0 \end{bmatrix} \le 0$$
(16)

for *j*, s=1,2,...,L, then the whole CACC system is stable. Proof. Let us define a Lyapunov function for the closed-loop CACC system (14) as,

$$V_{i}(t) = \sum_{i=1}^{N} x_{i}^{T}(t) P_{i} x_{i}(t)$$
(17)

The time derivative of  $V_i(t)$  is

$$\dot{V}_{i}(t) = \sum_{i=1}^{N} \dot{x}_{i}^{T}(t) P_{i} x_{i}(t) + x_{i}^{T}(t) P_{i} \dot{x}_{i}(t)$$
(18)

By substituting (14) into (18) with  $d_i(t) = 0$ , one can get

$$\dot{V}_{i}(t) = \sum_{i=1}^{N} \sum_{j=1}^{L} \sum_{s=1}^{L} h_{ij}(\theta_{i}(t)) h_{is}(\theta_{i}(t_{k}^{i}h)) \Big[ (A_{ij} + B_{ij}K_{is}) x_{i}(t) \Big]$$

$$+A_{i(i-1)j}x_{i-1}(t)^{T}P_{i}x_{i}(t)$$

+ 
$$x_i^T(t)P_i[(A_{ij} + B_{ij}K_{is})x_i(t) + A_{i(i-1)j}x_{i-1}(t)]$$

$$=\sum_{i=1}^{N}\sum_{j=1}^{L}\sum_{s=1}^{L}h_{ij}(\theta_{i}(t))h_{is}(\theta_{i}(t_{k}^{i}h))\left\{\begin{bmatrix}x_{i}(t)\\x_{i-1}(t)\end{bmatrix}^{\mathrm{T}}\right\}$$

$$\times \begin{bmatrix} (A_{ij} + B_{ij}K_{is})^T P_i + P_i(A_{ij} + B_{ij}K_{is}) & P_iA_{i(i-1)j} \\ A_{i(i-1)j}^T P_i & 0 \end{bmatrix} \begin{bmatrix} x_i(t) \\ x_{i-1}(t) \end{bmatrix}$$
(19)

Inequality (19) implies that  $\dot{V}_i(t) \le 0$ . Therefore the closed-loop CACC system (14) is asymptotically stable. This completes the proof.

Note that in the above discussions, we ignore the effects of disturbance  $d_i(t)$ , which may be the main causes of string instability and must be dealt with. The primary disturbances existing in a CACC system include the lead vehicle acceleration and wind gust. Here, we use  $d_i(t)$  to represent the combined equivalent disturbance in the CACCs. Then, we can reformulate the closed-loop CACCs in (14) as follows:

**Theorem 2.** For the closed-loop CACCs in (11), if  $P_i = P_i^T > 0$  is a common solution of the following linear matrix inequality

$$\begin{bmatrix} (A_{ij} + B_{ij}K_{is})^T P_i + P_i(A_{ij} + B_{ij}K_{is}) + Q_i & P_iA_{i(i-1)j} & P_i \\ A_{i(i-1)j}^T P_i & 0 & 0 \\ P_i & 0 & -\rho^2 I \end{bmatrix} \le 0$$
(20)

for *j*,*s*=1,2,...,*L*, then the whole CACC system is stable in the sense of Lyapunov if  $d_i(t)=0$  and the  $H_{\infty}$  performance in (15) is guaranteed for a prescribed  $\rho^2$ .

Proof. It's clear that (17) implies that  $\begin{bmatrix} (A_{ij} + B_{ij}K_{is})^T P_i + P_i(A_{ij} + B_{ij}K_{is}) & P_iA_{i(i-1)j} \\ A_{i(i-1)j}^T P_i & 0 \end{bmatrix} \le 0.$ 

According to Theorem 1, the stability of the whole interconnected nonlinear system is immediately followed. From (20), we obtain

$$\int_{0}^{t_{f}} x_{i}^{T}(t)Q_{i}x_{i}(t)dt = x_{i}^{T}(0)P_{i}x_{i}(0) - x_{i}^{T}(t_{f})P_{i}x_{i}(t_{f})$$
$$+ \int_{0}^{t_{f}} \{x_{i}^{T}(t)Q_{i}x_{i}(t) + \frac{d}{dt}(x_{i}^{T}(t)P_{i}x_{i}(t))\}dt$$

$$\leq x_{i}^{T}(0)P_{i}x_{i}(0) + \int_{0}^{t_{f}} \{x_{i}^{T}(t)Q_{i}x_{i}(t) + \dot{x}_{i}^{T}(t)P_{i}x_{i}(t) + x_{i}^{T}(t)P_{i}\dot{x}_{i}(t)\}dt = x_{i}^{T}(0)P_{i}x_{i}(0) + \sum_{j=1}^{L}\sum_{s=1}^{L}h_{ij}(\theta_{i}(t))h_{is}(\theta_{i}(t_{k}^{i}h))dt + \sum_{j=1}^{L}\sum_{s=1}^{L}h_{ij}(\theta_{i}(t))h_{is}(\theta_{i}(t_{k}^{i}h))dt + \sum_{j=1}^{L}\sum_{s=1}^{L}h_{ij}(\theta_{i}(t))h_{is}(\theta_{i}(t_{k}^{i}h))dt + \sum_{j=1}^{L}\sum_{s=1}^{L}h_{ij}(\theta_{i}(t))h_{is}(\theta_{i}(t_{k}^{i}h))dt + \sum_{j=1}^{L}\sum_{s=1}^{L}h_{ij}(\theta_{i}(t))h_{is}(\theta_{i}(t_{k}^{i}h))dt + \sum_{j=1}^{L}\sum_{s=1}^{L}h_{ij}(\theta_{i}(t))h_{is}(\theta_{i}(t_{k}^{i}h))dt + \sum_{j=1}^{L}\sum_{s=1}^{L}h_{ij}(\theta_{i}(t_{k}^{i}h))dt + \sum_{s=1}^{L}\sum_{s=1}^{L}h_{ij}(\theta_{i}(t_{k}^{i}h))dt + \sum_{s=1}^{L}h_{ij}(\theta_{i}(t_{k}^{i}h))dt + \sum_{s=1}^{L}h_{ij}(\theta_{i}(t_{k}^{i}h))dt + \sum_{s=1}^{L}h_{ij}(\theta_{i}(t_{k}^{i}h))dt + \sum_{s=1}^{L}h_{ij}(\theta_{i}(t_{k}^{i}h))dt + \sum_{s=1$$

$$+\{\int_{0}^{t_{f}} x_{i}^{T}(t)Q_{i}x_{i}(t) + \left[(A_{ij} + B_{ij}K_{is})x_{i}(t) + A_{i(i-1)j}x_{i-1}(t) + d_{i}(t)\right]^{T}$$

$$P_{i}x_{i}(t) + x_{i}^{T}(t)P_{i}\left[(A_{ij} + B_{ij}K_{is})x_{i}(t) + A_{i(i-1)j}x_{i-1}(t) + d_{i}(t)\right]dt = x_{i}^{T}(0)P_{i}x_{i}(0) + \sum_{j=1}^{L}\sum_{s=1}^{L}h_{ij}(\theta_{i}(t))h_{is}(\theta_{i}(t_{k}^{i}h))\left\{\int_{0}^{t_{f}} \left[x_{i}(t) + x_{i}^{T}(t) + d_{i}(t)\right]dt\right]dt = x_{i}^{T}(0)P_{i}x_{i}(0) + \sum_{j=1}^{L}\sum_{s=1}^{L}h_{ij}(\theta_{i}(t))h_{is}(\theta_{i}(t_{k}^{i}h))\left\{\int_{0}^{t_{f}} \left[x_{i}(t) + d_{i}(t)\right]dt\right]dt$$

$$\times \begin{bmatrix} (A_{ij} + B_{ij}K_{is})^T P_i + P_i(A_{ij} + B_{ij}K_{is}) + Q_i & P_iA_{i(i-1)j} & P_i \\ A_{i(i-1)j}^T P_i & 0 & 0 \\ P_i & 0 & -\rho^2 I \end{bmatrix}$$

$$\times \begin{bmatrix} x_i(t) \\ x_{i-1}(t) \\ d_i(t) \end{bmatrix} + \rho^2 d_i^T(t) d_i(t) \} dt$$
(21)

According to (20), we get

$$\int_{0}^{t_{f}} x_{i}^{T}(t)Q_{i}x_{i}(t)dt \leq x_{i}^{T}(0)P_{i}x_{i}(0) + \rho^{2} \int_{0}^{t_{f}} d_{i}^{T}(t)d_{i}(t)dt$$
(22)

Therefore, the  $H\infty$  control performance is achieved with a prescribed  $\rho^2$ . This completes the proof.

## 4. Simulation

In order to demonstrate the effectiveness of the proposed control method, we apply the proposed controller to CACCs which consists of one leader and two following vehicles, and runs in a virtual environment established by MATLAB. In the simulations, the parameters of the desired spacing is  $\delta_d = 1 \text{ m}$  and the length of the vehicle set to be  $L_i = 2\text{ m}$ , the time gap constant are chosen as h=0.8. The other parameters as follows:  $\sigma = 1m/s^3$ ,  $A_i = 2.2m^2$ ,  $c_{di} = 0.35$ ,  $m_i = 1500kg$ ,  $\tau_i = 0.2s$ ,  $\alpha = 0.5$  and  $d_i = 5$ .

The design procedure of the fuzzy state feedback controller for CACCs is given as follows:



Fig.1. Profile of the lead vehicle



Fig.2. Spacing errors

Step 1. To use the proposed fuzzy control method, we describe the interconnected CACCs by a fuzzy model. To simplify the design difficulty and complexity, we use as few rules as possible. Therefore, we approximate the CACCs by the following three-rule fuzzy model.

Rule 1 IF  $x_i(t)e_i$  is about 0, then

$$\dot{x}_{i}(t) = A_{i1}x_{i}(t) + B_{i1}u_{i}(t) + A_{i(i-1)1}x_{i-1}(t) + d_{i}(t)$$

Rule 2 IF  $x_i(t)e_i$  is about -0.5, then

$$\dot{x}_i(t) = A_{i2}x_i(t) + B_{i2}u_i(t) + A_{i(i-1)2}x_{i-1}(t) + d_i(t)$$

Rule 3 IF  $x_i(t)e_i$  is about 0.5, then

$$\dot{x}_i(t) = A_{i3}x_i(t) + B_{i3}u_i(t) + A_{i(i-1)3}x_{i-1}(t) + d_i(t)$$

where  $e_i$ ,  $A_{ij}$ ,  $B_{ij}$  and  $A_{i(i-1)j}$  are listed as follows

$$\begin{split} A_{11} &= A_{21} = A_{31} = \begin{bmatrix} 0 & -1 & 0 & -0.8 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0.5 & 0 \\ 0 & 0 & -44.100 & -5 \end{bmatrix}, \\ B_{i1} &= B_{i2} = B_{i3} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0.0033 \end{bmatrix}, e_i = \begin{bmatrix} 1 & 1 & 1 & 0 \end{bmatrix}, \\ A_{12} &= A_{22} = A_{32} = \begin{bmatrix} 0 & -1 & 0 & -0.8 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0.5 & 0 \\ 0 & 0 & -44.70 & -4.999 \end{bmatrix}, \\ A_{13} &= A_{23} = A_{33} = \begin{bmatrix} 0 & -1 & 0 & -0.8 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0.5 & 0 \\ 0 & 0 & -44.70 & -5.001 \end{bmatrix}, \\ A_{i(i-1)j} &= \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}. \end{split}$$

Step 2. To solve (20) by using LMI toolbox in Matlab. Step 3. The control gains  $K_{is}$  are the following:  $K_{i1} = [3.4210 \ 8.0156 \ 1.5072 \ 3.8065]$ ,

 $K_{i2} = \begin{bmatrix} 3.5620 & 8.0365 & 1.5152 & 3.8135 \end{bmatrix},$ 

 $K_{i3} = [3.4655 \ 8.0235 \ 1.5235 \ 3.8185].$ 



Fig.3. Velocities



Fig.4. Accelerations.

By using the aforementioned parameters for the CACCs, Fig. 2 and 3 was obtained, which show the maximum absolute spacing error and velocity are 1 m and 14 m/s, respectively, meaning the whole CACC system tracking accurately. Therefore, the simulation results demonstrate that the proposed  $H_{\infty}$  controller could guarantee the robust asymptotic stability of the fuzzy model CACC model in the event of external disturbances.

### **5.** Conclusion

In this research, a Takagi-Sugeno fuzzy model is proposed to study the CACC system control for interconnected nonlinearity vehicle dynamics using fuzzy model based  $H\infty$  control. The proposed controller can override the effect of external disturbance such that the asymptotic stability and string stability can be achieved at the same time. Simulations are given to illustrate the design procedure and shown that the proposed controller can achieve good performance.

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